Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International Advanced Level

Centre Number | Candidate Number

Monday 8 June 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. (a) Use the definition of $\sinh x$ in terms of exponentials to show that

$$\sinh 3x \equiv 4\sinh^3 x + 3\sinh x$$

(2)

(b) Hence determine the exact coordinates of the points of intersection of the curve with equation $y = \sinh 3x$ and the curve with equation $y = 19\sinh x$, giving your answers as simplified logarithms where necessary.

(5)

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Question 1 continued

Q1

(Total 7 marks)

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2. Determine

(i) $\displaystyle\int \frac{1}{3x^2 + 12x + 24}\,\mathrm{d}x$ **(4)**

(ii) $\displaystyle\int \frac{1}{\sqrt{27 - 6x - x^2}}\,\mathrm{d}x$ **(4)**

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Question 2 continued

Q2

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3.

$$\mathbf{M} = \begin{pmatrix} 3 & -4 & k \\ 1 & -2 & k \\ 1 & -5 & 5 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

Given that 3 is an eigenvalue of **M**,

(a) find the value of k.

(3)

(b) Hence find the other two eigenvalues of **M**.

(3)

(c) Find a normalised eigenvector corresponding to the eigenvalue 3

(3)

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Question 3 continued

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Question 3 continued

Q3

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4.

$$I_n = \int x^n \cos x \, \mathrm{d}x$$

(a) Show that, for $n \geqslant 2$

$$I_n = x^n \sin x + nx^{n-1} \cos x - n(n-1)I_{n-2}$$

(4)

(b) Hence find the functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ such that

$$\int x^4 \cos x \, \mathrm{d}x = \mathrm{f}(x)\sin x + \mathrm{g}(x)\cos x + c$$

where c is an arbitrary constant.

(5)

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Question 4 continued

Q4

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5. The hyperbola H has equation $\frac{x^2}{25} - \frac{y^2}{4} = 1$

The line l has equation $y = mx + c$, where m and c are constants.

Given that l is a tangent to H,

(a) show that $25m^2 = 4 + c^2$ **(4)**

(b) Hence find the equations of the tangents to H that pass through the point (1, 2). **(5)**

(c) Find the coordinates of the point of contact each of these tangents makes with H. **(3)**

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Question 5 continued

(Total 12 marks)

Q5

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6.

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & a \end{pmatrix} \quad a \neq 1$$

(a) Find $\mathbf{A}^{-1}$ in terms of a.

(4)

$$\mathbf{B} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}$$

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix **B**.

The equation of l_2 is

$$\left(\mathbf{r} - (12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})\right) \times (-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = \mathbf{0}$$

(b) Find a vector equation for the line l_1

(4)

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Question 6 continued

Q6

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7. The curve C has parametric equations

$$x = \cosh t + t, \qquad y = \cosh t - t \qquad 0 \leqslant t \leqslant \ln 3$$

(a) Show that

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 2\cosh^2 t$$

(3)

The curve C is rotated through 2π radians about the x-axis. The area of the curved surface generated is given by S.

(b) Show that

$$S = 2\pi\sqrt{2}\int_0^{\ln 3}\left(\cosh^2 t - t\cosh t\right)dt$$

(2)

(c) Hence find the value of S, giving your answer in the form

$$\frac{\pi\sqrt{2}}{9}\left(a + b\ln 3\right)$$

where a and b are constants to be determined.

(7)

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Question 7 continued

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8. The plane Π_1 has equation

$$x - 5y + 3z = 11$$

The plane Π_2 has equation

$$3x - 2y + 2z = 7$$

The planes Π_1 and Π_2 intersect in the line l.

(a) Find a vector equation for l, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

(5)

The point $P(2, 0, 3)$ lies on Π_1

The line m, which passes through P, is parallel to l.

The point $Q(3, 2, 1)$ lies on Π_2

The line n, which passes through Q, is also parallel to l.

(b) Find, in exact simplified form, the shortest distance between m and n.

(5)

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Q8

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TOTAL FOR PAPER: 75 MARKS

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